ISSN: 2036-5683 - EISSN: 2036-5462
Topologik - Rivista Internazionale di Scienze Filosofiche, Pedagogiche e Sociali/
Topologik - International Journal of Philosophy, Educational and Social Sciences
n. 15 /I semestre 2014 - Issue n° 15 / First semester 2014
Published By
LUIGI PELLEGRINI EDITORE

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Metatheoretical critics on current trends in Quantum Mechanics

Topologik
EISSN
2036-5462

Suggested citation for this article:
Aranda, C.C. (2014), «Metatheoretical critics on current trends in Quantum Mechanics», in *Topologik – Rivista Internazionale di Scienze Filosofiche, Pedagogiche e Sociali*, n. 15: 27-37;
URL: http://www.topologik.net/C._Aranda_Topologik_Issue_n.15_2014.pdf

Subject Area:
Philosophical Studies

Abstract

Is our purpose in this article to review several approaches to modern problems in quantum mechanics from a critical point of view using the approximation of the traditional mathematical thinking. Nevertheless we point out several natural questions that arise in abstract mathematical reasoning.

Keywords: mathematical physics, metamathematics, philosophical foundations, structural realism.

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1. Introduction.

In our approach we abuse of the resource of quotation, why? because we want to empathize several risks in current trends of quantum mechanics. Our tool will a careful comparison with several similar situations of the past and present in mathematical physics (G. Berkeley [3], T. V. Cao [7]). In opinion of several modern authors working on foundations of mathematics and physics (E. W. Beth page 3 [4] R. Penrose page 9 [16]), it is useful to remember a little of classical philosophy.

1.1. Quotations of Buddhist meditation.

Our main tool will be the use of some Buddhist insights, in words of V. K Rinpoche [17]:

Together, the five blessings, from oneself and the five through others comprise the ten blessings. Thus, to possess these, eighteen opportunities and blessings, forms the human birth. The difficulty of meeting with it is illustrated in three ways: by considering the cause, the numbers, and an example....Contemplation of impermanence in this way leads to comprehension of the impermanence of all composite things.....Thirdly, it is necessary to cultivate mindfulness of the failings of the cycle...
When the absorption deepens beyond these four, one experiences the Infinity of Space. If this absorption is practiced, one is born in the perception of the Infinity Space. Beyond this absorption, there are the perceptions the Infinity of Consciousness, Nothingness and Peak Cyclic Existence. In this state the subtle discrimination neither there nor not there, and one can be born as celestial beings in these states of perception. As the mind arrives on each succeeding level, there is successively more separation from attachment; thus mind becomes detached and penetrates to the next stage...

1.2. Aristotle's theory of science.

E. W. Beth consider of central importance in the foundations of mathematics a clear knowledge on the history and philosophical background. A nice analysis is perfomed in page 31 [4] where it stated that:

The essentials of Aristotle’s theory of science may be compressed into the following definition of the term ”deductive” or, as Aristotle says ”apodeictic” ”science”: A deductive science is a system S of sentences, which satisfies the following postulates:
(I) Any sentence belonging to S must refer to a specific domain of real entities;
(II) Any sentence belonging to S must be true;
(III) If certain sentences belong to S, any logical consequence of these sentences must belong to S;
(IV) There are in S a (finite) number of terms, such that (a) the meaning of these terms is so obvious as to require no further explanation; (b) any other term occurring in S is definable by means of these terms;
(V) There are in S a (finite) number of sentences, such that (a) the truth of these sentences is so obvious as to require no further proof; (b) the truth of any other sentence belonging to S may be established

by logical inference starting from these sentences.
The postulates (I), (II), and (III) will be called, respectively, the reality, the truth, and the deductivity postulate. The postulates (IV) and (V) together constitute the so-called evidence postulate; the fundamental terms and sentences, referred to in postulates (IV) and (V), are called the principles of the science under consideration. If all scientific knowledge is acquired by means of logical inference starting from a certain number of immediate, irreducible principles, which must be accepted as self-evident, then there arises, inevitably, the following anthropological question: whence do we, as human beings, obtain these principles, and in what manner may we account for our possession
and our use of them? This question has already been dealt with by Aristotle himself. In his opinion, we obtain the knowledge of the principles by way of an intuitive vision. This intuitive vision of the principles originates from induction on the basis of sense perception. This doctrine has an interesting theological-anthropological background, which should be mentioned here, as it has consequences of importance in the practice of scientific research.

Several modern scientist like N. Bohr argument that our brain in your deep chemical mechanism has a strong relationship with quantum principles, and therefore we pose a simple question, if this science and your development on intelligent creatures a natural law of living entities?, is this structure the unique possible in some senses?.

1.3. Brouwer's critics.

Perhaps our favourite tool was created by Brouwer (page 411,[4]):

In his criticism of Hilbert’s ideas, Brouwer gives a striking description of the successive stages in the formalization of mathematics. He enumerates:
(i) the construction of intuitive systems of mathematical entities;
(ii) the verbal parallel of mathematical thinking, that is, mathematical language;
(iii) the mathematical analysis of this language; this activity leads to the discovery of verbal edifices established in accordance with the principles of logic;
(iv) the step of abstracting from the meaning of the elements which constitute these verbal edifices; the abstract systems thus obtained are considered to be mathematical systems of the second order; they are
identical with the formal systems studied by symbolic logic;
(v) the introduction of the language of symbolic logic which accompanies logical constructions; this stage is found in the works of Peano and Russell;
(vi) the mathematical analysis of the language of logicians; this stage, initiated by Hilbert, had been neglected by Peano and Russell;
(vii) the step of abstracting ... etc.
According to Brouwer, mathematics is only to be found in the first stage of the process; the second stage in unavoidable from a practical point of view; the later stages are of a derivative character. In this analysis of the process of formalization we find a strikingly clear insight into the necessity of a separation between mathematics and metamathematics.

1.4. On the shape of mathematical arguments.

Aristotle's conception of science has a modern continuation with high degree of formality and simplicity, from A. J. M. Gasteren (page 10 [9]), we remark :

This chapter is concerned with some consequences of introducing nomenclature: repetitiousness, caused by the destruction of symmetry that is inherent to giving different things different names, and lack of disentanglement, caused by the availability of avoidable nomenclature. A second point the chapter wants to illustrate -and remedy- is how the use of pictures has the danger of strongly inviting:
(i) the introduction of too much nomenclature, and
(ii) implicitness about the justification of the steps of the argument.

(page 29 [9]) *This chapter discusses an extreme example of the harm done by the introduction of nomenclature that forces the making of avoidable distinctions, in particular the introduction of subscripted variables. In addition it illustrates some consequences of neglecting equivalence as a*
connective in its own right.

Nevertheless some modern tools like Quantum Field Theory use some metamathematical constructions like renormalization procedures, Feynman path integrals and they have remarkable success in experimental validation. Naturally several authors are involved in a effort to do almost an Aristotelic explanation of this science in the frontier of actual knowledge, for example: T. Muta [15], Zeidler [19, 20, 21] and the experimental validation, J. L. Basdevant and J. Dalivart [2]. Of course our interpretation of Aristotelic presentation are of doctrinal interpretation .

1.4.1. A critics of a modern mathematician from Aristotle's points of view.

Several branches of mathematics are involved with extreme technical development and with a serious lack of pedagogical explanations. Even outstanding mathematicians have a critical points of view on some current trends in mathematics, we appreciate in particular the preface of V. I. Arnold lectures on partial differential equations [1]:

In the mid-twentieth century the theory of partial differential equations was considered the summit of mathematics, both because of the difficulty and significance of the problems it solved and because it came into existence later than most areas of mathematics. Nowadays many are inclined to look disparagingly at this remarkable area of mathematics as an old-fashioned art of juggling inequalities or as a testing ground for applications of functional analysis. Courses in this subject have even disappeared from the obligatory program of many universities (for example, in Paris). Moreover, such remarkable textbooks as the classical three- volume work of Goursat have been removed as superfluous from the library of the University of Paris-7 (and only through my own intervention was it possible to save them, along with the lectures of Klein, Picard, Hermite, Darboux, Jordan, ...).
The cause of this degeneration of an important general mathematical theory into an endless stream of papers bearing titles like "On a property of a solution of a boundary-value

problem for an equation" is most likely the attempt to create a unified, all-encompassing, superabstract "theory of everything." The principal source of partial differential equations is found in the continuous-medium models of mathematical and theoretical physics. Attempts to extend the remarkable achievements of mathematical physics to systems that match its models only formally lead to complicated theories that are difficult to visualize as a whole, just as attempts to extend the geometry of second-order surfaces and the algebra of quadratic forms to objects of higher degrees quickly leads to the detritus of algebraic geometry with its discouraging hierarchy of complicated degeneracies and answers that can be computed only theoretically.

The situation is even worse in the theory of partial differential equations: here the difficulties of commutative algebraic geometry are inextricably bound up with non commutative differential algebra, in addition to which the topological and analytic problems that arise are profoundly nontrivial. At the same time, general physical principles and also general concepts such as energy, the variational principle, Huygen's principle, the Lagrangian, the Legendre transformation, the Hamiltonian, eigenvalues and eigenfunctions, wave-particle duality, dispersion relations, and fundamental solutions interact elegantly in numerous highly important problems of mathematical physics. The study of these problems motivated the development of large areas of mathematics such as the theory of Fourier series and integrals, functional analysis, algebraic geometry, symplectic and contact topology, the theory of asymptotics of integrals, microlocal analysis, the index theory of (pseudo-)differential operators, and so forth. Familiarity with these fundamental mathematical ideas is, in my view, absolutely essential for every working mathematician. The exclusion of them from the university mathematical curriculum, which has occurred and continues to occur in many Western universities under the influence of the axiomaticist/scholastics (who know nothing about applications and have no desire to know anything except the "abstract nonsense" of the algebraists) seems to me to be an extremely dangerous consequence of Bourbakization of both mathematics and its teaching. The effort to destroy this unnecessary scholastic pseudoscience is a natural and proper reaction of society (including scientific society) to the irresponsible and self-destructive aggressiveness of the "super-pure" mathematicians educated in the spirit of Hardy and Bourbaki.

This critics also hits the heart of several modern trends in physics like Quantum Gravity or String Theory, where we can see a kind of "super-pure" Bourbakization. We observe that the arguments of A. J. M. Gasteren [9] on the shape of mathematical arguments also contains a strong critics on Bourbakization. Moreover another victim of this preface of V. I. Arnold is a defense made by R. Penrose against the Popper's conception of science (the scientific admissibility of a proposed theory, namely that it be observationally refutable, page 1021 [16]). Even the Buddhist philosophy are in opposition with the defense made by R. Penrose, why he negates the Buddhist insights?.

2. Epistemology in the mathematical foundations.

M. Giaquinta in [10] gives an elegant description of the role played by philosophy in the foundations of mathematics. In several branches of physics, today this kind of foundation is a central problem [7, 8]. For our purposes is useful this quotation (page 620 [4]):

Starting from the methodological principles laid down in Tarski's semantics, Ajdukiewicz observes that these principles may be transferred by analogy from the domain of the logic of language to the parallel domain of epistemology. He then states, by way of supposition, the following thesis: it is possible in epistemology to use statements about thoughts to get conclusions about things thought of in these thoughts only under the condition that the language used by the epistemologist contains, from the beginning, not only the names of thoughts but also expressions denoting the things which are the objects of these thoughts. In the light of this thesis, based on analogy with the logic of language, a philosopher working epistemological problems has the choice between two alternatives.
(1) He may limit the language used in the course of his epistemological reflection to the language of syntax, understood in a broad sense, that is to a language containing solely names for the expressions of the object language, or names of thoughts which constitute the meanings of these expressions. A philosopher starting with such a limited language will not be able provided the thesis stated by Ajdukiewicz is true to resolve any problems of the object-language, and this means that he will not be
able to say anything whatever about the objects of the knowledge for which he is building a theory.
(2) He may use from the beginning of his reflection a language containing the object-language. In this case he will be, of course, obliged to conform to the syntactical and semantical rules of this language, that is, he is forced to solve all questions stated in the object-language by applying exactly the same methods as a man whose interests are limited to the object-world and who has no interest at all in epistemological problems.
A philosopher who chooses the second alternative must, on account of current word usage, be called a realist. This means that his manner of expressing himself reveals his belief which he shares with other scientific workers who use the object-language that houses, trees, mountains etc. exist in the literal sense of the word, that is, in the sense in which these words should be used on account of the syntactical and semantical rules for the object-language.

In a very elementary approach the authors want to raise again several points: This concepts are linked to the human entities by a kind of natural law?, in a mental experiment suppose that the mankind in a future disappears and another living entities are able to raise a technological civilization, this hypothetical civilization will be developed similar mathematical sciences structures or philosophical structures?

3. The experience as a natural driver of science development.

In words of Felix Klein: *The greatest mathematicians, such as Archimedes, Newton, and Gauss, always united theory and applications in equal measure.* In functional analysis a branch of modern mathematics, the theory is fundamentally motivated by differential equations. A detailed account of applications is offered by E. Zeidler in [18] and we reproduce here:

Numerous questions in physics, chemistry, biology, and economics lead to nonlinear problems; for example, deformation of rods, plates, and shells; behavior of plastic materials; surface waves of fluids; flows around objects in fluids or gases; shock waves in gases;

movement of viscous fluids; equilibrium forms of rotating fluids in astrophysics; determination of the shape of the earth through gravitational measurements; behavior of magnetic fields of astrophysical objects; melting processes; chemical reactions; heat radiation; processes in nuclear reactors; nonlinear oscillation in physics, chemistry, and biology; existence and stability of periodic and quasiperiodic orbits in celestial mechanics; stability of physical, chemical, biological, ecological, and economic processes; diffusion processes in physics, chemistry, and biology; processes with entropy production, and self-organization of systems in physics, chemistry, and biology; study of the electrical potential variation in the heart through measurements on the body surface to prevent heart attacks; determining material constants or material laws (e.g., coefficients of partial differential equations) from experimental data (inverse problems); gravitational effects of masses in the context of general relativity, gravitational collapse, black holes, big bang, and cosmological models; spectra of molecules, considering quantum mechanical electron interaction; determining molecular properties in quantum chemistry; statistical behavior of many particle systems; e.g., superconductivity, ferromagnets, and phase transitions (quantum statistics); behavior of plasma from the perspective of statistical physics and magnetohydrodynamics; behavior of lasers (quantum electronics); scattering processes of elementary particles and interactions of quantum fields (quantum field theory); unified theory of elementary particles (strong interaction and quarks, unification of weak and electromagnetic interactions, etc, gauge field theories); stochastic particle creation; e.g., through cosmic radiation, chain reactions in nuclear reactors, spreading of bacteria in epidemics, or waiting queues in economics (differential equations on convex sets for Markov chains); optimal control of processes; e.g., guiding missiles with minimal fuel consumption; stabilization of space platforms, moon landings, and returning space ships to earth; game-theoretic models in economics; modeling and optimizing production processes; optimizing stochastic processes; e.g., water and energy supplies; programming languages and interval arithmetic for computers (applications of the Tarski fixed-point theorem). As a rule, there arise nonlinear differential and integral equations, variational problems for integral expressions, and more general optimization problems.

Zeidler also gives a detailed list of objectives of this branch of mathematics:

Nonlinear functional analysis examines the following questions for each of the prototypes (l)-(5):
(i) Existence of a solution;
(ii) Uniqueness of a solution;
(iii) Stability of solutions under small perturbations of parameters;
(iv) Structure of the solutions set in the absence of uniqueness, e.g., branching of the solutions;
(v) Solvability conditions, e.g., conditions on y in (lb);
(vi) Construction of approximation methods, examination of their convergence, and acquisition of error estimates;
(vii) Justification of the linearization principle.
The significance of these examinations for the natural sciences becomes clear once we recall that the models used there are always of an approximate character, since it is not possible to take full account of all influences. The mathematical analysis of the models therefore carries

the obligation to decide on the utility of the models. At the very least, one should demand the existence of solutions. For heuristic reasons, one knows in many cases that with suitable auxiliary conditions, the procedure will follow a unique path. Then the model must have a unique solution. In general, uniqueness conclusions are more easily obtained than existence conclusions.

We remark the simplicity of the objectives and the constant interaction with problem facing for engineers:

In examining a model, special consideration must be given to stability questions. One must always be aware that a plain existence proof is of little utility to the natural scientist or engineer, since it is not all
solutions, but only the stable ones, that are ordinarily realized in nature. Unstable solutions can lead to technical or ecological catastrophes. Thus the duty of mathematics is not simply to provide quantitative results, but also to provide qualitative conclusions on the structure of the solutions. For example, for an engineer considering an oscillatory system, it is frequently of much greater importance to know that for long time periods the system is in stable equilibrium or is approaching a stable periodic solution (limit cycle) and to know these solutions, than to have an exact computation of short-term behavior (i.e., of transient oscillations). Furthermore, in dimensioning technical apparatus, the engineer must take care that the parameters do not fall into dangerous instability regions. In investigating the qualitative aspects of a solution, topological methods are particularly helpful.

4. The apriori knowledge and your validation from experience.

Applications to the Motion of the Perihelion of Mercury: *The last month was one of the most exciting and exhausting periods in my life, but also one of the most successful.... I recognized that my previous*
field equations of gravity were completely wrong. The Christoffel symbols have to be regarded as the natural expression for the components of the gravitational field.... The beautiful thing I experienced, was that not only Newton's theory could be obtained in first-order approximation, but also the motion of the Perihelion of Mercury in second-order approximation. For the deflection of light at the sun one finds a value twice as large as before. Einstein in a letter to Sommerfeld on November 28,1915.

We note that the experimental validation eliminates “Bourbakization” of a scientific stream. A beautiful example is the work of J. L Basdevant and J. Dalivart [2] on quantum mechanics and quantum field theory.

5. The computing problem.

In the actual context of high demands of energy three dimensional calculations on magnetohydrodynamics equations in Tokamaks becomes a big source of mathematical problems with concrete applications. This area of studies is founded in a long series of research papers [6] but this train of thoughts is not suffering of “Bourbakization” because the experimental constraint eliminates this problem. According to R. Schneider and R. Kleiber (page 199 [12]):

Physics in plasmas is determined in most cases by simple equations, namely the equations of motions for electrons, ions and neutrals including the effect of collisions and (self consistent) electric and magnetic fields. An exact numerical model based on individual particles is nevertheless impossible due to the larger number of particles involved and would need much too large computer resources. The simulation of plasma physics using of the most powerful computers available started in the fifties. Modeling fusion plasmas is inherently difficult due to the need to include a range of space scales extending from the gyro radius of the ions (a few mm) and electrons (a few 10^{-5} *m to the machine size (a few m) and the range of time scales extending from* 10^{-12} *s for fast electrons to several seconds for steady-state discharges. To cover a good part of these ranges in a computational model for a fussion plasma about 10 physical variables are necessary (densities, velocities, temperature). These have to be updated at approximately* 10^{10} *grid points for* 10^{8} *time steps. Assuming* 10^{2} *operations for a very simple numerical algorithm per grid point and time step this results in a total number of* 10^{21} *floating point operations (FLOP). The execution speed of a code on a computer is usually measured in mega-flops (*10^{6} *FLOP/s), giga-flops (*10^{9} *FLOP/s) or tera-flops (*10^{12} *FLOP/s). Assuming a realistic performance of 60 giga-flops for such a model (Cray T3E with 512 processors in parallel), one run would take some 500 years of computer time. Therefore, the brute force ansatz will not work and a hierarchy of models is necessary. The task of computational plasma physics is to develop such that methods in order to obtain a better understanding of plasma physics. For this, a close contact to theoretical plasma physics methods is necessary.*

For a review of the state of the art in numerical methods in plasma physics and applications to Tokamaks see the works of J. Blum [5] and S. Jardin [11]. For a better comprehension of the difficulties posed by real calculation we remember (S. G. Mikhlin pages XII and XIII [14]):

Comparatively quickly, however serious shortcomings in the variational method approach appeared: the difficulty of coordinate functions, satisfying given boundary conditions, for many complex forms of the region, and of forming the Ritz system in connection with the fact that its coefficients and right hand side values are usually expressed by integrals the evaluation of which, particularly for two or more independent variables, can require a large expenditure of labour. This last fact did not have, it is true, serious significance as long as only comparatively crude approximations, consisting of linear combination of a small number of coordinate functions, were constructed. Along with the variational method, finite difference methods were developed. At first, the theoretical significance of these methods was, perhaps, greater than practical: for hand calculations, the solution of systems of sufficiently high order, to which finite difference methods led, were often unrealizable. The appearance of computers changed the situation. Important advantages of finite difference methods over variational became apparent: for finite difference methods, the form of the region is of little importance and the formation of difference systems is based on a simple and similar rules. At present, difference methods are used considerably more often than variational. At this point, it is important to note the basic limitations of difference methods.

The laboriousness of the solution of systems of high dimension continues to manifest itself. In Forsythe and Wasow, written in 1959, approximate data on the amount of machine time necessary for the solution of the usual problems in mathematical physics by finite difference methods was given. Assuming that the execution of one arithmetic operation requires 50 microseconds of machine time, Forsythe and Wasow concluded: in order to obtain a solution with high accuracy, one dimensional problems require 1 hour, two dimensional requires 6 weeks and three dimensional require 100 years of machine time. Of course, with the appearance of faster machines the situation has improved. Nevertheless, it is probable that for multidimensional problems, the application of finite difference methods will still lead to an excessive outlay of machine time.

Also a full description of numerical challenges is given by A. N. Tikhonov and V. Y. Arsenin [13].

6. Final comments.

Using a very simple approximation we raise several elementary questions about the nature of mathematics, metamathematics and the utilization in the experience materialized in physical experiments. There exists the possibility of mathematics and metamathematics are determined by some kind of natural laws and in the light of this possibility that we call the reality has a weak epistemological foundation in the sense that our thoughts are constrained to some eternal law and we cannot across this frame to discover new tools for an more advanced understanding of information produced by the experience. Also domains of mathematics are suffering failing cycles like phenomena in the Buddhist context: the “Bourbakization” is opposed to Aristotle’s approach of simplicity: the truth of these sentences is so obvious as to require no further proof. And this leads to a simple consideration, a mathematical model of some phenomena necessarily needs to be able to offer qualitative or quantitative answers. If a model is too complex for offer answers in our train of thoughts has a very limited value and this collides with several current trends in physics.

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